

# EVOLUTIONARY DESIGN OF CORRUGATED HORN ANTENNAS

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**Abstract.** An evolutionary programming (EP) algorithm is used to optimize pattern of a corrugated circular horn subject to various constraints on return loss and antenna beamwidth and pattern circularity and low cross-polarization. The EP algorithm uses a Gaussian mutation operator. Examples on design synthesis of a 45 section corrugated horn, with a total of 90 optimization parameters, are presented. The results show excellent and efficient optimization of the desired horn parameters.

## 1. Introduction

Corrugated horn antennas are frequently used as the feed elements in ground-based reflector antennas for satellite and deep space communications. In particular, for the latter application, it is desirable to design a horn that has a nearly perfect circularly symmetric pattern (i.e., identical E- and H-plane patterns), with zero or low cross-polarization, and at the same time achieves a specified beamwidth and a low return loss at the design frequency range. A parametric study of the corrugated horns, however, shows that the objective function relating the pattern shape, beamwidth and  $S_{11}$  is a nonlinear function of the corrugation dimensions and has many local optima. As a result one has to resort to a global optimization technique such as Evolutionary Algorithms for a successful design of these antennas.

The Evolutionary Algorithms (EAs) are in general multi-agent stochastic search methods that rely on a set of variation operators to generate new offspring population. A selection scheme is then used to probabilistically advance better solutions to the next generation and eliminate less-fit solution according to the objective function being optimized. Among, the three paradigms of EAs, namely, Genetic Algorithms (GA), Evolutionary Programming (EP) and Evolution Strategies (ES), GA and EP have been successfully applied to the design and optimization of various antenna and microwave structures [1,2,3,4].

The variation operator used in GA is a combination of crossover and mutation with the former being the main mechanism of change. The selection of the crossover and mutation probabilities is rather arbitrary and they are not adapted during evolution. EP, however, models the evolution at the species level, thus its variation operator is entirely based on mutation where adaptive and/or self-adaptive techniques exist for adapting the parameters of mutation operator during the evolution process. Mutation-based reproduction process in EP, coupled with the fact that unlike the conventional GA, EP works directly with continuous parameters, provides a versatile tool in design of the problem specific variation operators for multi-parameter antenna optimization, and easy integration with available apriori knowledge about the problem.

In this work we have used an EP algorithm with a Gaussian mutation operator to optimize pattern of a corrugated circular horn subject to various constraints on return loss and antenna beamwidth. A software code, based on generalized scattering matrix and mode-matching technique [5], which has been developed and modified at JPL over many years, is used for the analysis of the corrugated horn and the calculation of the radiated far field from the horn. Examples on design synthesis of a 45 section corrugated horn, with a total of 90 optimization parameters, are presented. The initial design for the horn, as shown in Figure 1, is generated using various guidelines in the literature [6,7] and by implementing a simple MATLAB program.

## 2. Application of EP to Design of Corrugated Horns

For optimization purposes the N-section corrugated horn in Figure 1 is mathematically represented as a vector of length  $n = 2N$ :

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$$\bar{X} = [r_1, r_2, \dots, r_N; d_1, d_2, \dots, d_N]^T \quad (1)$$

where  $r_i$  and  $d_i$  are radius and length of the  $i$ -th corrugated segment, respectively. For the minimization of the difference between the E- and the H-plane co-polar patterns, subject to the constraints on the beamwidth and the overall return loss, we construct the fitness function as,

$$F(\bar{X}) = \frac{1}{M_\theta} \sum_{i=1}^{M_\theta} [E(\theta_i, 0^\circ) - E(\theta_i, 90^\circ)] + \alpha_1 [E(\theta_b, 0^\circ) - q_1] + \alpha_2 [q_2 - E(\theta_b, 0^\circ)] + \beta (S_{11} - S_{opt}) \quad (2)$$

with

$$\alpha_1 = \begin{cases} w_1 / M_\theta & \text{if } q_1 \leq E(\theta_b, 0^\circ) \\ 0 & \text{otherwise} \end{cases} ; \alpha_2 = \begin{cases} w_2 / M_\theta & \text{if } E(\theta_b, 0^\circ) \leq q_2 \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

in which  $E(\theta, \phi)$  is the normalized co-polar pattern in dB obtained by the application of the mode matching code already mentioned..  $M_\theta$  in the first term is the number of elevation angles,  $\theta$ , sampled in the interval  $[0, \theta_{\max}]$ , in which we require a near circularly symmetric pattern. The second and third terms in (2) penalize all the solutions that violate the constraint,  $q_2 \leq E(\theta_b, 0^\circ) \leq q_1$ , on the normalized gain value at  $\theta_b$ , while the last term penalizes those that violate the constraint,  $S_{11} \leq S_{opt}$ , on the return loss. The weighting factors  $w_1$ ,  $w_2$  and  $\beta$  are selected such that to prioritize the influence of various constraints on the fitness function.

To optimize vector  $\bar{X}$  in (1), we apply a continuous parameter meta-EP. The process consists of five basic steps: initialization, fitness evaluation, mutation, tournament and selection [8,9]. In particular, an initial population of  $\mu$  individuals is formed through a uniform random or a biased distribution. We consider each individual to be a pair of real-valued vectors,  $(\bar{x}_i, \bar{\eta}_i)$ ,  $\forall i \in \{1, \dots, \mu\}$  where  $\bar{x}_i = [x_i(1), x_i(2), \dots, x_i(n)]$  and  $\bar{\eta}_i$  are the  $n$ -dimensional solution and its corresponding strategy parameter (variance) vectors, respectively.  $\bar{\eta}_i$ 's are initialized based on the user-specified search domains,  $\bar{x}_i \in \{x_{\min}, x_{\max}\}^n$ , which may be imposed at this stage. A meta-EP algorithm is now implemented by generating a single offspring  $(\bar{x}_i', \bar{\eta}_i')$  from each parent  $(\bar{x}_i, \bar{\eta}_i)$  according to:

$$x_i'(j) = x_i(j) + \sqrt{\eta_i(j)} N_j(0, 1) \quad ; \quad \eta_i'(j) = \eta_i(j) e^{[\tau' N(0, 1) + \tau N_j(0, 1)]} \quad (4)$$

for  $j = 0, 1, 2, \dots, n$ , where  $x(j)$  and  $\eta(j)$  are the  $j$ th components of the solution vector and the variance vector, respectively.  $N(0, 1)$  denotes a one-dimensional random variable with a Gaussian distribution of mean zero and standard deviation one.  $N_j(0, 1)$  indicates that the random variable is generated anew for each value of  $j$ . The scale factors  $\tau$  and  $\tau'$  are commonly set to  $(\sqrt{2\sqrt{n}})^{-1}$  and  $(\sqrt{2n})^{-1}$ , respectively, where  $n$  is the dimension of the search space mechanism for generating new offspring population. The details on the other steps of the meta-EP process can be found in [4,8,9].

### 3. Numerical Results

As an example, we have optimized a 45-segment corrugated horn. The geometry of the initial structure, before optimization, is shown in Figure 1. The population size and the number of opponents in the tournament selection of EP were set to  $\mu = 10$  and  $q = 4$ , respectively. To optimize the  $X$  vector in (1), the radii and lengths of the sections were randomly initialized in the search domains  $0.95r_{0i} < r_i < 1.05r_{0i}$  and  $0.7d_{0i} < d_i < 1.3d_{0i}$ , where  $r_{0i}$  and  $d_{0i}$  are the radius and length of the  $i$ -th segment of the initial structure, respectively. The strategy parameters were initialized to  $(x_{\max} - x_{\min})/6$  and kept above a lower bound of  $10^{-4}$  during the self-adaptations in (4). Two cases are considered here. In each case 200 generations were performed.

In Case I, the optimization was performed subject to the constraint on the return loss ( $< -40$  dB) only, i.e.  $\alpha = 0$  in (2), and with  $M_\theta = 60^\circ$ , corresponding to a near circular symmetric pattern up to an elevation angle of 60 degrees. The Fitness-value trajectory of the best overall population member for Case I is shown in Figure 2. The optimization was performed at  $f = 8$  GHz. Figure 3 and 4 show the E- and H-plane far-field patterns and the difference between them, respectively. The corresponding patterns for the initial structure are also included for

comparison. Figure 5 shows the  $S_{11}$  versus frequency for the initial as well as the optimized horn. As can be seen the optimization has resulted in a return loss of about  $-50$  dB and an almost perfect circularly symmetric pattern at the design frequency. The geometry of the final optimized structure is shown in Figure 6. The narrow band of the optimized design at 8GHz and the "non-smooth" variation of the corrugations can be attributed to the fact that the original design has a better match and bandwidth around 9.5 GHz while optimization is performed around 8 GHz. Optimization around 9.5 will result in a much wider bandwidth as well as a smoother variation of the corrugations.

In Case II, the optimization was performed at  $f = 9.5$  GHz. In this case the pattern was optimized with  $M_0 = 45$ , subject to the constraints of  $S_{11} < -40$ dB and the normalized field,  $-21\text{dB} \leq E(\theta_p = 45^\circ, \phi^\circ) \leq -19\text{dB}$ . Figures 7 shows the E- and H-plane far-field patterns of the optimized horn while Figure 8 shows the frequency variation of  $S_{11}$ .

#### 4. Conclusions and future work

In this paper we have shown that the evolutionary programming techniques can be successfully applied to the problem of corrugated horn design. Many additional results will be shown at the conference. The next step already being worked on, is the parallelization of the optimization program on a massively parallel computer such as SGI Origin 2000 at JPL. The present work was performed on the JPL CRAY SV1 and takes many hours of computation which can be substantially reduced by parallelization. The parallel code will be applied to the design multi-frequency X/Ka feed horns of the JPL/NASA Deep Space Network (DSN) reflector antennas.

#### 5. Acknowledgment

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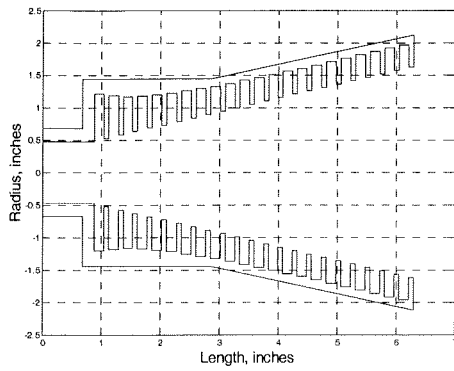


Figure 1. Geometry of corrugated horn before optimization

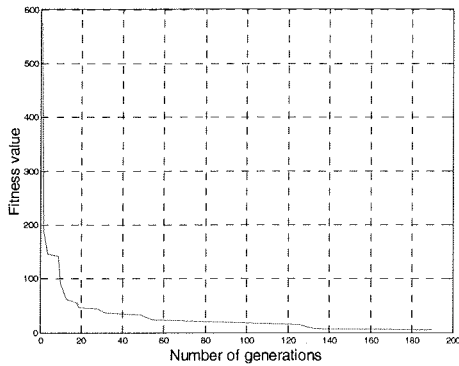


Figure 2. Trajectory of the fitness-value of the best overall population member, Case I.

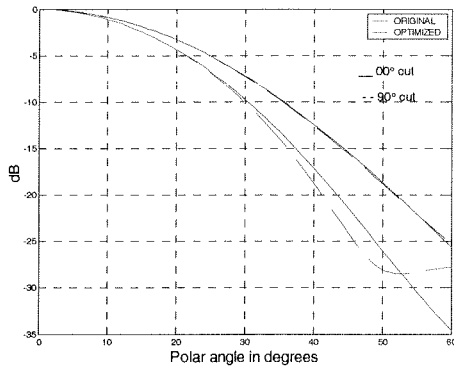


Figure 3. Comparison of horn patterns in two orthogonal planes before & after optimization, Case I.

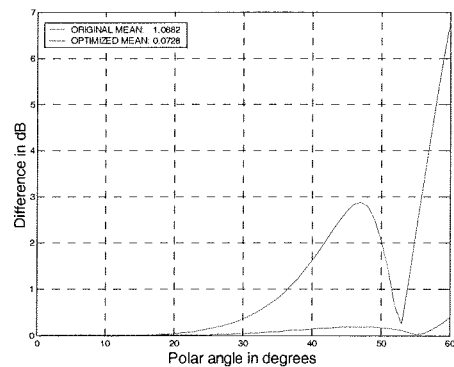


Figure 4. Pattern difference in the two planes before & after optimization, Case I.

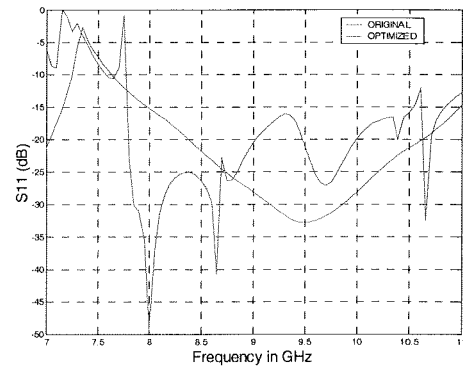


Figure 5. Input  $S_{11}$  as a function of frequency for the horn before and after optimization at 8GHz, Case I.

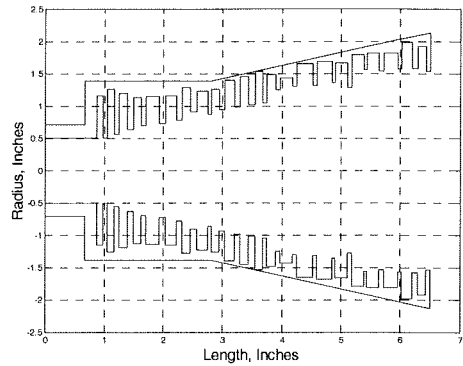


Figure 6. Geometry of corrugated horn after optimization, Case I.

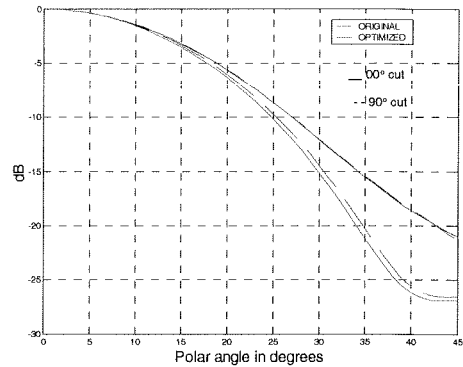


Figure 7. Comparison of horn patterns in two orthogonal planes before & after optimization, Case II.

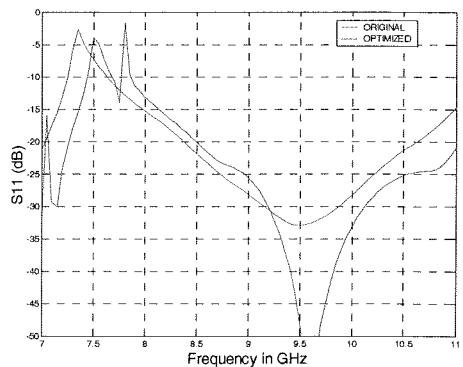


Figure 8. Input  $S_{11}$  as a function of frequency for the horn before & after optimization at 9.5 GHz, Case II.